MATH3070 Introduction to Topology 2017-2018

Tutorial Classwork 9

1. Prove that $\mathbb{S}^1$ is not a retract of $\mathbb{D}^2 = \{z \in \mathbb{C} \mid |z| \leq 1\}$.

2. Consider the torus $\mathbb{S}^1 \times \mathbb{S}^1$. Show that the circle $\mathbb{S}^1 \times \{1\}$ is not a deformation retract of the torus.

3. (a) Denote the n-dimensional disk by $\mathbb{D}^n = \{\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid x_1^2 + x_2^2 + \cdots + x_n^2 \leq 1\}$. Show that $\mathbb{S}^{n-1}$ is a deformation retract of $\mathbb{D}^n \backslash \{0\}$.

   (b) * Hence, show that $\mathbb{D}^2$ is not homeomorphic to $\mathbb{D}^n$ for $n > 2$.

   (You may assume the fact that for any $n > 2$ and any $\mathbf{x} \in \mathbb{D}^n$, there exists a homeomorphism $f : \mathbb{D}^n \to \mathbb{D}^n$ which maps the point $\mathbf{x}$ to 0.)